Theoretical analysis of a polarisation switching VCSEL used as a Doppler velocimeter

Miguel C. Soriano (1), Jan Albert (1), Irina Veretennicoff (1), Krassimir Panajotov (1,2)

(1) Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussel (2) Permanent address: Institute of Solid State Physics, 72 Tzarigradsko Chuassee Blvd. 1784 Sofia, Bulgaria

Abstract

Semiconductor lasers can be used as Doppler velocimeters. Reflection of their light on a moving object causes a frequency shift and when the light is fed back into the cavity, the self-mixing effect induces a modulation of the output power. The frequency of this modulation is proportional to the speed of the moving object. It has been shown experimentally at the UCC (University of Cork, Ireland) [1] that the sensitivity of such a sensor is increased when using polarisation switching vertical-cavity surface-emitting lasers (VCSELs). In this work we present a theoretical intensity rate equation model that explains well the experimental observations.

Introduction

Laser Doppler velocimetry is a non-intrusive optical method for measuring the velocity $v$ of an object. Its principle is based on the Doppler effect: when an electromagnetic wave is reflected on a moving object a frequency shift proportional to $v$ is induced. Measurement of this frequency shift gives access to the speed of the object. Here this is done by feeding the scattered light back into the laser cavity. The self-mixing effect [2], [3] then causes a modulation of the output power with a frequency that is in its turn proportional to $v$. An example of a Laser Doppler velocimeter is shown in fig. 1.

![Figure 1: A small portion of the light reflected from the moving object is fed back into the laser cavity and is mixed with the original wave oscillating inside the laser.](image-url)

Vertical-cavity surface-emitting lasers (VCSELs) are being used successfully in a wide variety of applications. The polarisation behaviour of VCSELs is somewhat peculiar. Even in the single (transverse) mode regime two orthogonal linear polarisation modes (PMs) are supported and switching between them can occur. Experiments have shown [1] that the responsivity of VCSELs used as Doppler velocimeters is highly increased when working in the vicinity of a polarisation switching (PS) point.

In a first section we introduce a set of a rate-equations that could explain the observed increase in sensitivity. In the second we present numerical simulations and compare these with the experimental results.
The model

We describe the polarisation switching VCSEL with a set of intensity rate equations: one equation for the intensities of each of the PMs and one for the charge carrier density. This approach has been studied before and we refer to [4] for further information. To take optical feedback into account we follow the Lang-Kobayashi approach [5] and include the Doppler frequency shift $\omega_D$ into the feedback terms.

Two further assumptions are made: (i) The optical feedback is weak, such that single external cavity mode operation [2], [6] is guaranteed. (ii) The electric field intensity changes little in an external cavity round-trip time. Both assumptions have been verified numerically. Taking the latter into account and after some simple manipulations, the dimensionless equations read:

$$\frac{dp_x}{dt} = \rho \left[ g_x (1 - e_{sx} p_x - e_{sy} p_y) n - 1 + \frac{2 \kappa \tau_p}{\tau_m} \cos(\omega_D \tau - \Delta \phi(\tau)) \right] p_x + \beta (1 + n) \left[ 1 + \frac{\kappa \tau_p}{\tau_m} \cos(\omega_D \tau - \Delta \phi(\tau)) \right]$$

$$\frac{dp_y}{dt} = \rho \left[ g_y (1 - e_{sy} p_y - e_{sx} p_x) n - 1 + \frac{2 \kappa \tau_p}{\tau_m} \cos(\omega_D \tau - \Delta \phi(\tau)) \right] p_y + \beta (1 + n) \left[ 1 + \frac{\kappa \tau_p}{\tau_m} \cos(\omega_D \tau - \Delta \phi(\tau)) \right]$$

$$\frac{dn}{dt} = j - n - n(1 - e_{sx} p_x - e_{sy} p_y)p_x - n(1 - e_{sy} p_y - e_{sx} p_x)p_y$$

Here $p_{x,y}(t)$ denote the dimensionless photon numbers; $n(t)$ is the reduced carrier number; $g_{x,y}$ are the dimensionless gain coefficients; $\rho$ is the reduced injected current; $\beta$ is the reduced spontaneous emission term; $\rho = \tau_p / \tau_s$ where $\tau_p$ and $\tau_s$ are the photon and carrier lifetimes, respectively; $\kappa = (1 - r_2^2)(r_{ext} / r_2)$ is the feedback coefficient and $r_2$, $r_{ext}$ denote the facet and external mirror reflection coefficients, respectively; $2 \kappa \tau_p / \tau_m \equiv \kappa_1$ and $\kappa \tau / \tau_m \equiv \kappa_2$ serve as reduced feedback coefficients; $\omega_D = \omega_d \frac{\tau}{\tau_s}$ is the reduced Doppler shift; $\Delta \phi(\tau) = \omega_F \tau + \omega_d \tau$ is the $\tau$ dependent phase term; $\omega_d = 2 \pi (2v / \lambda)$ is the Doppler frequency shift; $\omega_F$ denotes the frequency in the presence of feedback; $e_{ij} = \frac{E_{ij}}{E_{j0} r_{NJ}}$ are the reduced gain saturation coefficients, where $G_{NJ}$ is the differential gain of the $j$-mode; $\tau_{in}$ is the laser cavity round trip time; $\tau = 2L_{ext} / c$ is the external cavity round-trip time, where $L_{ext}$ is the length of the external cavity. Effects of self- and cross-saturation between the PMs have been included. These give rise to switching through a bistable region [4].

Numerical simulations and comparison with experiments

It can be shown that, under the assumption we made, the effect of the Doppler shifted feedback is equivalent to a harmonic modulation of the net gain, and thus of the output.
power \[7\]. This can be easily understood if one realizes that the electric field inside the cavity is, roughly speaking, nothing but the superposition of two harmonic waves of slightly different frequencies. In \[7\] we show that if the laser driving current \(j\) is biased inside the bistable region, the modulation of the gains of the two PMs will cause PS, provided the reduced feedback strength \(\kappa_1 = \frac{2\pi\tau_p}{\kappa_2}\) exceeds \(\Delta j\), where \(\Delta j\) is the width of the bistable region. This case is shown in (fig. 3a). If the feedback strength is too weak the switching does not take place (fig. 3c). This last case is equivalent to case of intensity modulation only. It is plain to see that the modulation depth of the polarisation resolved output of (fig. 3a) is much bigger than the one of (fig. 3c). A spectral analysis confirms the enhancement of the sensitivity: see figures. 3b-3d.

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\begin{align*}
\rho_x &= 10^{-3}, g_x = 7, g_y = 7.02, \beta_{xy} = 10^{-5}, e_{xx} = 0.003, e_{xy} = 0.003, e_{yx} = 0.0049, e_{yy} = 0.005, \Delta \phi(\tau) = 0, \omega_D = 0.003, \kappa_2 = 0.5, j = 0.35. \\
P(\text{a, c}) \text{ Intensities of the 2 PMs versus time and (b, d) the power spectrum of the y-PM: eqs.(1-3) with parameters: } & \\
\text{In figures 4 a & b we show the polarisation resolved traces as recorded by P. Porta et al. [1]. Comparison of figures 3a-b and 4a-b shows that the agreement between our model and the experimental observations is excellent.}
\end{align*}
\]

**Conclusions**

In this contribution we have paved the way to a theoretical analysis of VCSELs used as Doppler velocimeters. Experimental results [1] have demonstrated that the sensitivity of VCSELs in this application is highly increased when working in the vicinity of a PS point. We have used an intensity rate equation model, consisting of two photon density equations (one for each PM) and one charge carriers equation. Effects of self- and cross-saturation between the PMs were included. The Doppler-shifted feedback was included in
the equations following the standard Lang-Kobayashi approach. We have shown numerically that when the VCSEL is biased inside the bistable region the modulation caused by the self-mixing effect is enough to make the polarisation switch periodically between the two orthogonal modes. In this way the polarisation resolved output shows an increased modulation amplitude. Indeed: the switching is between the on and the off state of a mode. The correspondence between the numerical results obtained with our model and the experimental data [1] was checked in the time as well as the frequency domain and turned out to be excellent.

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References


