Frequency comb generation in a time-dependent graphene ribbon array

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Graphene ribbon lattices are known for the high tunability of their plasmonic resonances. These lattices couple light very efficiently into compact plasmonic modes. Here we use a time modulated graphene lattice that allows plasmonic resonances and modulations to interact and generate frequency combs from a monochromatic pulse. We use numerical simulations and coupled mode theory to connect the frequency comb features to the resonance and applied modulation properties. This mechanism permits efficient and highly tunable frequency comb generation ranging from the mid-infrared to the far-infrared.

Introduction

Graphene plasmonics has several very attractive features: high tunability of the plasmonic modes, strong confinement and relatively low losses [1, 2]. Graphene arrays of nanoribbons exhibit interesting properties induced by plasmonic resonances [3] leading to coherent perfect absorption [4] among others. In this paper we extend the concept introduced by Ginis et al. [5] for comb generation: when a plane wave impinges onto a graphene sheet with a time-dependent conductivity, frequency combs are generated in transmission. Here we go one step further by using time-dependent graphene lattices and the interaction with plasmonic resonances. We show that this comb generation principle is highly tunable using a simple Coupled Mode Theory (CMT) model and Finite Element Method (FEM) simulations.

Graphene ribbon arrays

We study a graphene nanoribbon lattice under normal incidence (Figure 1a). The incoming light couples with the plasmonic modes of the grating (which is not the case for a graphene sheet because of the phase mismatch). In order to model this structure for 2D FEM simulations, we describe the graphene by a current line with a Drude-like conductivity [6] (Equation 1).

\[
\sigma(\omega) = \frac{e^2 E_F}{\pi \hbar^2 \omega} \frac{-j}{\omega - j\tau_{gra}^{-1}}
\]

Where \(E_F\) is the (adjustable) graphene doping level, \(\omega\) is the angular frequency and \(\tau_{gra} = 10\) ps is the electron relaxation time in graphene. For simplicity we use a dispersionless model where \(\omega\) is set to \(2\pi \times 10^{13}\) rad/s (we work around that frequency in the following sections). The transmittance spectrum of this structure (Figure 1b) exhibits dips that correspond to plasmonic modes of the grating.

In order to efficiently model these plasmonic resonances, we use a CMT model (Figure 2a) that consists of a resonant cavity with a mode amplitude \(a(t)\) (normalized so...
Figure 1: (a) Representation of the incoming TM wave and reference axes. Grating parameters are the width of the ribbon $D$ and the grating period $L$. (b) Transmittance, reflectance and absorptance spectra for $L = 10 \, \mu m$, $D = 8.75 \, \mu m$ and $E_F = 0.635 \, eV$.

Figure 2: (a) Schema of the CMT model used. (b) FEM simulation (dots) and CMT fit (solid lines) of a plasmonic resonance in the graphene grating.

that $|a(t)|^2$ is the energy in the cavity) coupled to two ports $s_{1,2\pm}(t)$ (normalized so that $|s_{1,2\pm}(t)|^2$ is the power flowing through the port). The equations for this model are:

$$\frac{da(t)}{dt} = \left\{ j\omega_{res} \left[ E_F(t) \right] - \frac{1}{\tau_{tot}} \right\} a(t) + \kappa s_{1+}(t) \quad (2)$$

$$s_{2-}(t) = d a(t) + j e^{\phi} f s_{1+}(t) \quad (3)$$

where $\omega_{res}$ is the resonance frequency of the cavity, $f$ is the direct transmission without a cavity, $\phi$ is a phase connected to $f$ and $\kappa$. $1/\tau_{tot} = 2/\tau + 1/\tau_{abs}$ is the total decay rate of the cavity and $1/\tau_{abs}$ is the absorption rate of the cavity.

This model can also describe a cavity with a time-dependent resonance frequency $\omega_{res}[E_F(t)]$ that is obtained by varying the graphene doping level in time. Equation 2 then becomes:

$$\frac{da(t)}{dt} = \left\{ j\omega_{res}[E_F(t)] - \frac{1}{\tau_{tot}} \right\} a(t) + \kappa s_{1+}(t) \quad (4)$$
This equation along with Equation 3 can describe frequency comb generation in graphene gratings. When a quasi-monochromatic pulse (Figure 3a) with form:

\[ s_{1+}(t) = \exp\left[-\frac{(t-t_0)^2}{q^2}\right] \exp\left[j\omega_0(t-t_0)\right] \]  

is sent through port \( s_{1+} \), it generates a frequency comb in the output port \( s_{2-} \). Here \( q \) gives the width of the pulse, \( \omega_0 \) is the central pulse angular frequency, \( t_0 \) is the pulse center time. The modulation we use for \( E_F \) has the form:

\[ E_F(t) = \frac{E_{F\text{max}} - E_{F\text{min}}}{2} \sin(\omega_{\text{mod}}t) + \frac{E_{F\text{max}} + E_{F\text{min}}}{2} \]  

where \( \omega_{\text{mod}} \) is the modulation frequency, \( E_{F\text{min}} \) and \( E_{F\text{max}} \) are the minimum and maximum doping levels during the modulation. Using these expressions, we can solve the CMT equations and compare them with FEM simulation.

**Comparison FEM and CMT**

We use a graphene grating with \( L = 10 \mu\text{m} \) and \( D = 8.75 \mu\text{m} \), a modulation with \( E_{F\text{min}} = 0.62 \text{ eV} \), \( E_{F\text{max}} = 0.65 \text{ eV} \) and \( \omega_{\text{mod}} = 2\pi \times 10^{11} \text{ rad/s} \), and a pulse with \( \omega_0 = 2\pi \times 9.8 \times 10^{12} \text{ rad/s} \) and \( q = 1250/\omega_0 \). In Figure 3b we compare results from the CMT model and FEM simulations both in time and frequency domains using these parameters. The two methods are in excellent agreement, allowing us to validate the CMT model.

**Link between grating and comb properties**

In this section we link the properties of the grating (indicating the CMT parameters) and the modulation parameters to the generated frequency comb shape. The frequency of the modulation gives the spacing between the frequency components in the comb (Figure 4a). The parameter \( \tau \) (linked to the cavity lifetime) also influences the comb shape (Figure 4b). For a small \( \tau \), light couples efficiently to the cavity but also outcouples efficiently leading to a limited side frequency generation (light does not stay long in the cavity). For longer \( \tau \), the cavity does not couple efficiently with the incoming light and the comb generation mechanism is less efficient.
Figure 4: (a) Frequency combs obtained for different modulations frequencies $f_{\text{mod}}$. Frequency components in the combs are always separated exactly by $f_{\text{mod}}$ (b) Frequency combs obtained for different $\tau$ values (the $\tau$ axis is in logarithmic scale).

This mechanism can scale from mid-infrared to far-infrared by choosing the relevant grating parameters so that the resonance frequency of the plasmonic mode corresponds to the required wavelength range where the comb needs to be generated. The combs produced by this method are thus highly tunable.

**Conclusion**

We exploit the interaction between light, plasmonic resonances and time-modulation to generate frequency combs from a nearly monochromatic pulse. We give a CMT model that describes the frequency comb generation mechanism and that reproduces with great accuracy the results from FEM simulations. This model allows to link the combs shape to the grating and time-modulation parameters: the spacing between the frequency components of the comb is always equal to the modulation frequency and the coupling between light and the cavity influences the efficiency of the comb generation. This comb generation principle is highly tunable and can work from mid-infrared to far-infrared frequencies.

**References**


